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13. ABSTRACT (Maximum 200 words) The gravitational perturbations are ever present and to counter their effect requires fuel, and spacecraft have to carry all their fuel. Thus, the guiding principle of our program is "Use, don't fight Kepler". Our approach has not been to not just treat this as a control problem but to utilize the physics to find the orbits that require the least amount of fuel to control and then develop the control techniques to minimize fuel consumption. Using this approach we have identified the non-resonant relative motion orbits that remain close together without thrusting in the presence of the J2 perturbation. Some of the relative orbits obtained for high inclination orbits of the chief satellite are too large for practical use. We have determined alternative specifications that will result in mission specific relative orbits, albeit at the expense of a small amount of fuel. We have developed a state transition matrix for elliptic orbits under the influence of J2. We have developed novel feedback control strategies, unique to this problem. A formation maintenance strategy that consumption has been complete. We have also developed a strategy for formation resizing using effects into the selection of the initial conditions for relative motion orbits. This effort is now complete and this report is the final report.					
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# **FINAL REPORT**

## **ANALYSIS AND DISTRIBUTED CONTROL OF A FORMATION OF INTELLIGENT SATELLITES**

F49620-99-1-0075

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### **Objectives**

1. Develop mathematical models for the dynamics and control of relative motions of swarms of satellites of a Distributed Satellite System under the influence of gravitational perturbations; differential atmospheric drag and nonlinear effects.
2. Solve the nonlinear celestial mechanics dynamical system problem of finding the initial conditions that will result in the desired relative motion.
3. Develop the optimal control strategies and architectures that will maintain the desired configuration with minimum fuel and maximize the time interval between maneuvers. Further, develop optimal rotational slewing strategies for dwelling on targets within the system constraints.
4. Develop solutions for providing the relative position and attitude information necessary to achieve the desired control and mission requirements. Develop algorithms for measurement and sensing based on novel miniature electro-optical sensors.

### **Status of Effort**

The gravitational perturbations are ever present and to counter their effect requires fuel, and spacecraft have to carry all their fuel. Thus, the guiding principle of our program is "Use, don't fight Kepler". Our approach has not been to not just treat this as a control problem but to utilize the physics to find the orbits that require the least amount of fuel to control and then develop the control techniques to minimize fuel consumption. Using this approach we have identified the non-resonant relative motion orbits that remain close together without thrusting in the presence of the  $J_2$  perturbation. Some of the relative orbits obtained for high inclination orbits of the chief satellite are too large for practical use. We have determined alternative specifications that will result in mission-specific relative orbits, albeit at the expense of a small amount of fuel. We have developed a state transition matrix for elliptic orbits under the influence of  $J_2$ . We have developed novel feedback control strategies, unique to this problem. A formation maintenance strategy that will result in uniform fuel consumption among all the satellites, as well as reduced overall fuel consumption has been completed. We have also developed a strategy for formation resizing using a dynamics approach. We have developed a method for incorporating some of the nonlinear effects into the selection of the initial conditions for relative motion orbits.

This effort is now complete and this report is the final report.

## Accomplishments

### J2 INVARIANT ORBITS

With relative motion about a spherically symmetric Earth the only constraint necessary for periodic relative motion is that the two orbits have equal energy, i.e., equal semi-major axes. When the gravitational perturbations are included the equal energy orbit constraint is no longer sufficient for bounded relative motion; differential nodal precession, differential perigee precession and in-track drift can occur. To prevent all three of these drifts the two orbits must have equal energy, angular momentum and inclination. These three constraints severely restrict the relative motion orbits options. With the  $J_2$  Invariant Orbit approach we have identified a class of relative motion orbits that have minimal drift. First, we have developed the constraint, called the "period matching constraint", that prevents in-track drift and replaces the equal semi-major axis or equal energy constraint. This constraint is  $\delta\dot{M} + \delta\dot{\omega} + \delta\dot{\Omega} \cos i = 0$ , where  $M$  is the mean anomaly,  $\omega$  is the argument of perigee and  $\Omega$  is the right ascension and  $i$  is the chief orbit inclination. This is the primary constraint that needs to be enforced. An additional constraint can be applied to prevent out of plane drift. These two constraints yield the  $J_2$  Invariant Orbits. In this research we have also shown that relative motion orbits should be designed in mean element space, not osculating element space. In mean element space the critical elements affecting the drift of the orbits are the three momenta variables,  $(a, e, i)$ . In osculating space all elements affect the drift and design is much more difficult.

### STATE TRANSITION MATRIX

Hill's equations assume a spherical earth and a circular reference orbit. The gravitational perturbations create a change in orbit period, nodal precession, perigee drift and short and long period variations of the other orbit elements. Consequently, Hill's equations do not provide a good estimate of the long term relative motion needed for minimum fuel control. If Hill's equations are used the control system will misinterpret the cause of some of the relative motion and make incorrect orbit adjusts. For example, the short period relative motion due to the equatorial bulge  $J_2$  could be interpreted as a secular drift and a mismatch in semi-major axis, and consequently make a semi-major axis adjust that would create secular drift. Consequently, we need a better dynamic model for the relative motion. We have developed the state transition matrix for the linearized equations of relative motion when the reference orbit is elliptic and both satellites experience the effect of  $J_2$ . This state transition matrix has been developed without solving the equations of motion. We have developed a method called the "Geometric Method" that uses the relationship between the differences in the orbital elements and the relative state. This state transition matrix has been developed for both mean and osculating elements. With  $x$  as the difference in radii,  $y$  as the in-track curvilinear separation and  $z$  as the out of plane curvilinear separation,  $\mathbf{x} = (x, \dot{x}, y, \dot{y}, z, \dot{z})^T$  and  $\delta\mathbf{e}$  as the difference in the orbital elements of the chief and deputy we can derive

$$\mathbf{x} = \Sigma \delta\mathbf{e}, \Sigma = (A + \alpha B), \alpha = 3J_2 R_c^2 \quad (1)$$

$(\mathbf{x}, \mathbf{e})$  denote the osculating elements and the matrices  $(A, B)$  are a function of these elements. From orbit theories we can obtain the solution of the time rate of change of the mean elements and the relationship between the mean and osculating elements. Thus,

$$\begin{aligned} \delta\mathbf{e} &= D \delta\mathbf{e}_m, D = \frac{\partial \mathbf{e}}{\partial \mathbf{e}_m} = I + J_2 D^* \\ \delta\mathbf{e}_m &= \phi_{em} \delta\mathbf{e}_{m0} \end{aligned} \quad (2)$$

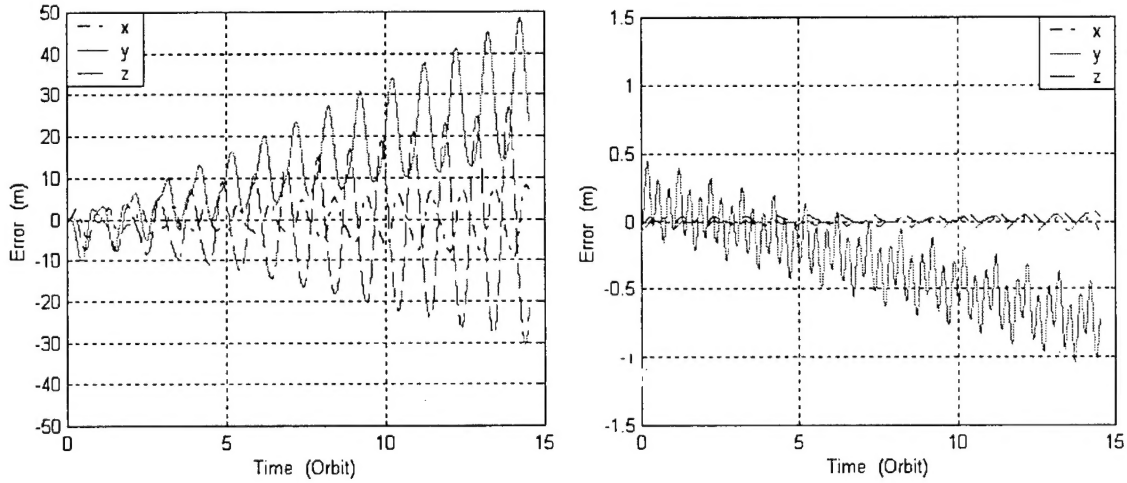
Thus,

$$\begin{aligned}
x &= \Phi x(t_0) \\
\Phi &= \Sigma D \phi_{em} D^{-1}(t_0) \Sigma^{-1}(t_0)
\end{aligned} \tag{3}$$

If we are using mean instead of osculating elements  $D$  is the identity matrix and

$$\Phi = \Phi_m = \Sigma_m \phi_m \Sigma_m^{-1}(t_0), \Sigma_m = (A_m + \alpha B_m) \tag{4}$$

The functional form of  $A_m$  is the same as  $A$ , but the elements are evaluated using mean elements rather than osculating elements. However, the matrix  $B$  is different because the angular velocity of the reference frame is different for osculating and mean elements. Using this state transition significantly improves the prediction of the long-term relative motion as shown in the following figure, which is for a near circular chief orbit with elements of ( $a = 7100 \text{ km}, e = 0.005, i = 70 \text{ deg}$ ) and the desired relative motion orbit is the horizontal plane circular orbit of radius 500 m. Hill's equations show a substantial secular error growth in the in-track direction and periodic errors in all three axes. The in-track error growth is due primarily to the unmodeled Chief orbit eccentricity. The Geometric Method produces errors several orders of magnitude smaller.



(a) Hill's Equations Errors      (b) With Osculating Elements  
Figure 2 Geometric Method Errors for Perturbed Near-Circular Orbit

## FUEL MINIMIZING/BALANCING FORMATION CONTROL

We have developed a novel formation control approach for reference orbits with small eccentricity by including and exploiting the  $J_2$  perturbation. The control effort required to counter the perturbation on each deputy is directly proportional to its inclination difference. Over a period of time, the deputy with maximum inclination difference will lose more mass due to fuel consumption than one with zero inclination difference. Mass imbalance between the satellites with the same geometric characteristics, will cause variations in the ballistic coefficients, and hence create differential drag among them. Differential drag is not a concern if all the satellites have the same ballistic coefficients. The key result is the identification of resonance terms in the

dynamics and introducing an optimal detuning parameter. The control action introduces periodic variations in all of the mean orbital elements of the deputies. A novel, disturbance accommodating control design process is utilized for accommodating short period oscillations in the dynamics and rejecting secular disturbances only. This architecture not only minimizes the total fuel consumption among all the satellites but also maintains the same, average fuel consumption of each satellite, over a desired period of time. Analytical solutions are derived for obtaining the initial conditions of the deputies to populate the desired relative orbit. The reference solution obtained from Hill's equations is modified to account for the eccentricity of the chief's orbit. The control concept has been verified by performing nonlinear simulations.

We have extended our analysis to the cases of general periodic orbits, special cases of which are the circular projection relative orbits (TechSat21) and Circular relative orbits (LISA). The general expression for the optimal formation rotation rate is of the form

$$\begin{aligned} x/R_0 &= 0.5c_1 \sin(n_0 t + \alpha) \\ y/R_0 &= c_1 \cos(n_0 t + \alpha) + c_3 \\ z/R_0 &= c_2 \cos(n_0 t + \alpha + \varphi) \end{aligned} \quad (5)$$

where  $R_0$  is the radius of the chief which is time varying for elliptic orbits. The size and shape of the relative orbit is specified by the four parameters  $c_1, c_2, c_3$ , and  $\varphi$ . The dynamic phase shift is introduced through the detuning parameter  $\alpha$ , the optimal value of which based upon minimizing a quadratic performance index is given below:

$$\dot{\alpha} = \frac{c_1^2 \bar{\varepsilon} - A c_2 \cos \varphi}{c_1^2 + c_2^2} \quad (6)$$

$$\text{where } \bar{\varepsilon} = -\varepsilon_1 J_2 \left( \frac{R_e}{a_0} \right)^2 n_0 \sin(2i_0) \quad (7)$$

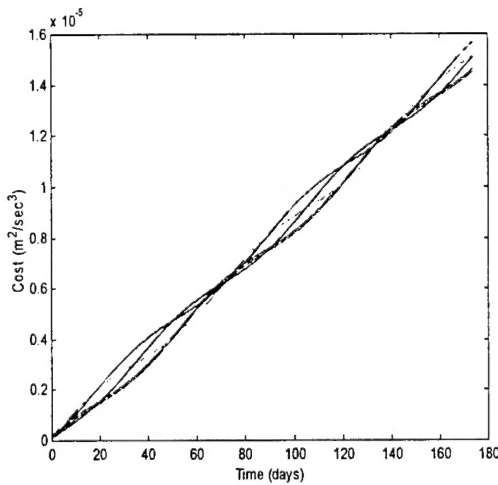


Figure 3 Cost Curves for the satellites with  $\dot{\alpha} = 0$ .

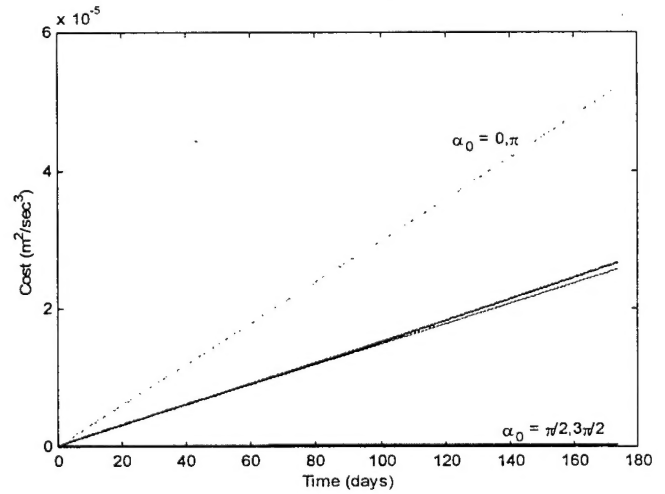


Figure 4 Cost Curves for the satellites with optimal  $\dot{\alpha}$

Figure 3 shows the cost for the formation of eight satellites, plotted as a function of  $\dot{\alpha}$  for  $i_0 = 70^\circ$ . This plot was obtained by integrating the mean square acceleration for each satellite over a period of one year. The fuel balancing nature of the controller is obvious from this figure. The optimality of the concept is clear from Fig. 4 which shows the control cost when  $\dot{\alpha} = 0$ . It should be kept in mind that the performance of the formation will degrade when the first satellite runs out of fuel. Figure 5 shows the radii of the circular projection orbits computed over a period of one year. The short period oscillations that have been accommodated by the controller can easily be discerned from this figure.

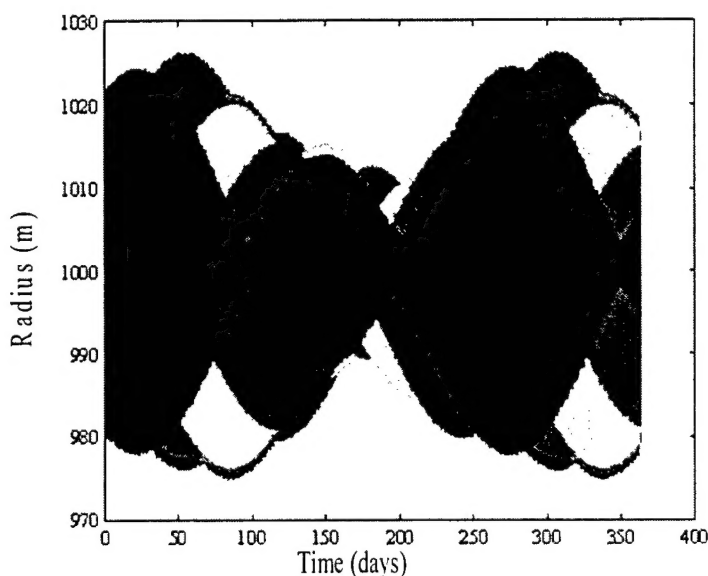
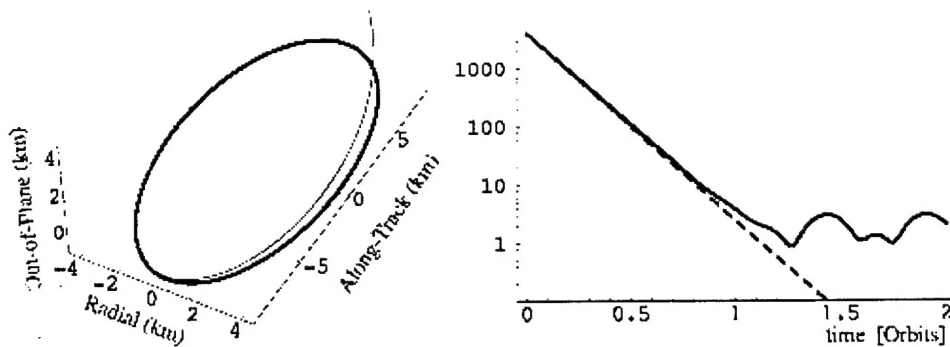


Figure 5 Radii of the Circular Projection Relative Orbits,  $i_0 = 70^\circ$

#### NONLINEAR FEEDBACK CONTROL:

A direct method for mapping orbit element differences to their corresponding local Cartesian coordinate differences has been developed. This mapping is used to construct a hybrid continuous feedback control law for achieving the desired relative orbit geometry, explicitly given in terms of orbit element differences. The actual desired orbit is specified in terms of local Cartesian coordinates. A numerical simulation illustrates the performance and limitations of such feedback control laws. Using the linearized mapping between the relative orbit coordinates, causes only a small performance penalty. However, it is advantageous to work in mean element space when determining the relative orbit tracking error. Figure 6 shows the performance of the hybrid control law in tracking a specific relative orbit (shown in Figure 6.a), where the  $J_2$  gravitational perturbation is neglected. The tracking error obtained using the linear mapping between Cartesian relative coordinates and orbit elements is shown as a solid line, while the tracking error if the nonlinear mapping is used is shown as a dashed line. Using the linear approximation, results in a steady state error of only 1 m.



a) Relative Orbit shown in Chief LVLH Frame      b) Tracking Error (m)

Fig 6 Simulation Results using Hybrid Feedback control

### Personnel Supported

K. T. Alfriend (Professor), S. R. Vadali (Professor), J. L. Junkins (Professor), D. W. Gim (PhD student), S. Vaddi (PhD student).

### Interactions

1. Alfriend gave a seminar entitled "Formation Flying Satellites: Control By an Astrodynamacist" at Ohio State, Stanford University, Georgia Tech and UCLA.
2. Alfriend visited Princeton Satellite Systems in Falls Church, VA to discuss TAMU's research and formation resizing.
3. Gim, D-W and Alfriend, K.T., "The State Transition Matrix of Relative Motion for the Perturbed Non-Circular Reference Orbit," Paper No. AAS 01-222, AAS/AIAA Space Flight Mechanics Conference, Santa Barbara, CA, Feb 11-16, 2001.
4. H. Schaub of Sandia National Labs has been interacting with Alfriend and Vadali.
5. Vadali, S.R., Alfriend, K.T. and Vaddi, S., A New Concept for Constellation Control of Formation Flying Satellites," Paper No. AAS 01-218, AAS/AIAA Space Flight Mechanics Conference, Santa Barbara, CA, Feb 11-16, 2001.
6. Alfriend, K.T., Vadali, S.R. and Schaub, H., Celestial Mechanics Aspects of Formation Flying Satellites," presented at the Division of Dynamical Astronomy Annual Meeting, Houston, TX, April 23-25, 2001.
7. Alfriend, K.T. and Vadali, S.R., "Constellation Control of Formation Flying Satellites," presented at the SIAM Conference on Control, San Diego, CA, July 11-14, 2000.
8. Vadali, S.R., Schaub, H., and Alfriend, K. T., "Initial Conditions and Fuel-Optimal Control for Formation Flying of Satellites," 1999 GNC Conference, AIAA Paper 99-4265, Portland, OR, August 99.
9. Vadali, S.R., Vaddi, S., Naik, K., and Alfriend, K. T., "Control of Satellite Formations," Paper No. AIAA 2001-4028, AIAA GNC Conference, Montreal, Canada, August 6-10, 2001.
10. Vaddi, S.S., Vadali, S.R. and Alfriend, K.T., "Approximate Models for Spacecraft Relative Motion Analysis, Paper No. AAS 02-184, 2002 AAS/AIAA Space Flight Mechanics Conference, San Antonio, January 26-29, 2002.
11. Gim, D.-W. and Alfriend, K.T., "The State Transition Matrix For Relative Motion of Formation Flying Satellites," Paper No. AAS 02-186, 2002 AAS/AIAA Space Flight Mechanics Conference, San Antonio, January 26-29, 2002.

### Transitions

1. Some of the algorithms developed in this project are to be used for the formation control of TechSat 21. Point of contact: Dr. Craig McLaughlin.
2. Some of our research in formation flying has been transitioned to the Power Sail project of AFRL. Point of contact: Robert Howard.

### Honors/Awards

#### During grant:

1. Alfriend, K.T. elected to the National Academy of Engineering, 1999.
2. Junkins, J.L., IAF Frank J. Malina Astronautics Medal, awarded October 1999.
3. Paper 3 under Interactions was selected as the Best Paper for the 2001 AAS/AIAA Space Flight Mechanics Conference.
4. Paper No. 5 under Interactions was selected as the runner-up for the Best Graduate Student Paper in the Department of Aerospace Engineering for 2001

#### Prior to grant:

1. Alfriend, K.T. Fellow AIAA (1988), Fellow AAS (1985), Fellow International Academy of Astronautics (1998), AIAA Mechanics and Control of Flight Award (1998), AAS Dirk Brouwer Award (1989)
2. Junkins, J.L. National Academy of Engineering (1996), Fellow AIAA (1987), Fellow AAS (1984), International Academy of Astronautics (1996), AIAA Mechanics and Control of Flight Award (1983), AAS Dirk Brouwer Award (1987), ASEE/AIAA Atwood Award (1990), AIAA Pendray Aerospace Literature Award (1990), AIAA von Karman Medal and Lectureship (1997).

### Publications

1. Schaub, H and Alfriend, K.T., " $J_2$  Invariant Relative Orbits for Formation Flying," *International Journal of Celestial Mechanics and Dynamical Astronomy*, Vol. 79, 2001, pp 77-95.
2. Schaub, H., Vadali, S.R., Junkins, J.L., and Alfriend, K.T., "Spacecraft Formation Flying Control Using Mean Orbit Elements," *Journal of Astronautical Sciences*, Vol. 48, No. 1, January-March 2000, pgs 69-87.
3. Alfriend, K. T. and Schaub, H., "Dynamics and Control of Spacecraft Formations: Challenges and Some Solutions," *AAS J. Of the Astronautical Sciences*, Vol. 48, No. 1, January-March 2000, pgs 69-87.
4. Alfriend, K. T., Schaub, H., and Gim, D.-W., "Gravitational Perturbations, Nonlinearity and Circular Orbit Assumption Effect on Formation Flying Control Strategies," accepted for publication in *AAS J. Of the Astronautical Sciences*, Vol.
5. Schaub, H. and Alfriend, K. T., "Impulsive Spacecraft Formation Flying Control to Establish Specific Mean Orbit Elements," *AIAA J of Guidance, Control and Dynamics*, Vol. 24, No. 4, July-August 2001.
6. Alfriend, K.T. and Schaub, H., "Hybrid Coordinate and Orbit Element Feedback Law for Formation Flying Spacecraft," to appear in *AIAA J. of Guidance, Control and Dynamics*.
8. Vadali, S.R., Vaddi S. S., and Alfriend, K. T., "A New Concept for Controlling Formation Flying Satellite Constellations," to appear in the *AIAA J. of Guidance, Control, and Dynamics*.
9. Vadali, S.R., Vaddi, S.S. and Alfriend, K.T., "An Intelligent Control Concept for Formation Flying Satellite Constellations," to appear in the *Journal of Nonlinear Control*.
10. Gim, D-W and Alfriend, K.T., "The State Transition Matrix of Relative Motion for the Perturbed Non-Circular Reference Orbit," submitted to the *Journal of Guidance, Control, and Dynamics*.